| CSE 392: Matrix and Tensor Algorithms for Data | Spring 2024 |
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| **Homework 4** | |
| | *Due Date: 04-19-2024* |

*Assignments are to be submitted through Canvas, and should be individual work. You can discuss the problems, but should submit individually. Preferably typewritten.*

### Problem 1. Coherence

The coherence $\mu(\boldsymbol{A})$ of $\boldsymbol{A} \in \mathbb{R}^{N \times r}$ is the *maximum leverage score* of $\boldsymbol{A}$. Prove that:

1. $\mu(\boldsymbol{A} \otimes \boldsymbol{B}) = \mu(\boldsymbol{A})\mu(\boldsymbol{B})$

2. $\mu(\boldsymbol{A} \odot \boldsymbol{B}) \leq \mu(\boldsymbol{A})\mu(\boldsymbol{B})$

### Problem 2. Tucker-ALS

Download the Monkey BMI data from (`https://gitlab.com/tensors/tensor_data_monkey_bmi`). The `data.mat` contains a 3-way tensor of size $43 \times 200 \times 88$. You have used this dataset in HW3 Problem 5.

(a) Run and time Tucker ALS for ranks 5:5:20. I.e., use the same rank for all three factor matrices. Plot the relative errors.

(You can use `tucker_als` function from tensor toolbox or `tucker` function from tensorly package)

(b) Compare the relative errors and the runtimes you get for tucker ALS against those you got for CP ALS (in HW3 P5) for the same set of ranks. What are the tradeoffs between the two methods?

(c) Try varying the ranks for the different factor matrices in Tucker ALS. Can we achieve better compression for fixed storage using different ranks for the factor matrices?

In Matlab, you will have to apply `tensor` function to convert matlab array to a tensor object.

### Problem 3. tSVD

Use the Monkey BMI data again (`https://gitlab.com/tensors/tensor_data_monkey_bmi`).

(a) Run and time t-SVD for ranks 5:5:20. Plot the relative errors.

(You can use the Matlab or python functions provided in the class)

(b) Compare the relative errors achieved by CP, Tucker, and t-SVD decompositions for a given rank. What are the trade offs?

(c) Try applying tSVD to different orientations of the tensor.

(d) Try t-SVDM with a DCT matrix.